GCE

Mathematics

Advanced GCE

Unit **4725:** Further Pure Mathematics 1

Mark Scheme for January 2011

Oxford Cambridge and RSA Examinations

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1	**(i)**	$(7 \quad 9)$	B1B1 **2**	Each element correct **SC** (7,9) scores B1
	(ii)	(18)	B1* depB1 **2**	Obtain correct value Clearly given as a matrix
	(iii)	$\begin{pmatrix} 12 & -4 \\ 6 & -2 \end{pmatrix}$	M1 A1 A1 **3** **7**	Obtain 2×2 matrix Obtain 2 correct elements Obtain other 2 correct elements
2.	**(i)**	$-12+13\mathrm{i}$	B1B1 **2**	Real and imaginary parts correct
	(ii)	$\frac{27}{37}-\frac{14}{37}\mathrm{i}$	B1 M1 A1 A1 **4** **6**	z^* seen Multiply by w^* Obtain correct real part or numerator Obtain correct imaginary part or denom. Sufficient working must be shown
3			B1* M1* A1* depA1 **4** **4**	Establish result true for $n = 1$ or 2 Use given result in recurrence relation in a relevant way Obtain $2^n + 1$ correctly Specific statement of induction conclusion
4		***Either*** $\frac{a}{4}n^2(n+1)^2+\frac{bn}{2}(n+1)$ $a=4 \quad b=-4$ ***Or*** $a+b=0 \quad 4a+b=12$ $a=4 \quad b=-4$	B1 M1 A1 M1 A1 A1 **6** M1 A1 A1 M1 A1 A1 **6**	Correct value for $\sum r$ stated or used Express as sum of two series Obtain correct unsimplified answer Compare coefficients or substitute values for n Obtain correct answers Use 2 values for n Obtain correct equations Solve simultaneous equations Obtain correct answers
5		$\mathbf{A}^2$	B1 M1 A1cao **3** **3**	$(\mathbf{A}^{-1})^{-1} = \mathbf{A}$ seen or implied Use product inverse correctly Obtain correct answer

6	(i)	(a)	B1*		Vertical line
			depB1	**2**	Clearly through (4, 0)
		(b)	B1		Sloping line with +ve slope
			B1		Through (0, -2)
			B1ft	**3**	Half line starting on y-axis 45° shown convincingly
	(ii)		B1ft		Shaded to left of their **(i) (a)**
			B1ft		Shaded below their **(i) (b)** must be +ve slope
			B1ft	**3**	Shaded above horizontal through their (0, -2) **NB** These 3 marks are independent, but 3/3 only for fully correct answer.
				8	

7	(i)	$\begin{pmatrix} 1 & 3 \\ 0 & 1 \end{pmatrix}$	B1 B1	**2**	Each column correct
	(ii)		B1*		Enlargement or stretch in x **and** y axes
			depB1	**2**	Scale factor $\sqrt{3}$
	(iii) (a)		B1		$(2,0),(6,2)$ indicated
			B1		(8, 2) seen
			B1	**3**	Accurate diagram, including unit square
	(b)	det**C** = 4	B1		Correct value found
			B1	**2**	Scale factor for area
				9	

8	(i)	***Either***			
		$\alpha+\beta=\frac{1}{2}, \alpha\beta=\frac{3}{2}$	B1		State or use both correct results in (i) or (ii)
		$\alpha+\beta+\frac{\alpha+\beta}{\alpha\beta}$ or $\alpha+\beta+\frac{2}{3}(\alpha+\beta)$	M1		Express sum of new roots in terms of $\alpha+\beta$ and $\alpha\beta$
			M1		Substitute their values into their expression
		$p=\frac{5}{6}$	A1	**4**	Obtain **given** answer correctly
		Or			
		$3u^2-u+2(=0)$	B1		Substitute $x=\frac{1}{u}$ and obtain correct quadratic (equation)
			M1		Use sum of roots of new equation
			M1		Substitute their values into their expression
		$p=\frac{5}{6}$	A1		Obtain **given** answer correctly

(ii)	$\alpha'\beta' = \alpha\beta + \frac{1}{\alpha\beta} + \frac{\beta}{\alpha} + \frac{\alpha}{\beta}$	B1		Correct expansion
	$\frac{\beta}{\alpha} + \frac{\alpha}{\beta} = \frac{(\alpha+\beta)^2 - 2\alpha\beta}{\alpha\beta}$	M1		Show how to deal with $\alpha^2 + \beta^2$
		A1		Obtain correct expression
	$q = \frac{1}{3}$	M1		Substitute their values into $\alpha'\beta'$
		A1	**5**	Obtain correct answer a.e.f.
		9		
9 (i)		M1		Show correct expansion process for 3 x 3
		M1		Correct evaluation of any 2 x 2
	det **M** = $a^2 - 7a + 6$	A1	**3**	correct answer
(ii)		M1		Solve det**M** = 0
	a = 1 or 6	A1A1	**3**	Obtain correct answer, ft their (i)
(iii)		M1		Attempt to eliminate one variable
		A1		Obtain 2 correct equations in 2 unknowns
		A1	**3**	Justify infinite number of solutions **SC** 3/3 if unique solution conclusion consistent with their (i) or (ii)
		9		
10 (i)		M1		Use correct denominator
		A1	**2**	Obtain **given** answer correctly
(ii)		M1		Express terms as differences using (i)
		M1		Do this for at least 3 terms
		A1		First 3 terms all correct
		A1		Last 2 terms all correct
	$\frac{1}{2} - \frac{1}{n+1} + \frac{1}{n+2}$	M1		Show relevant cancelling
		A1	**6**	Obtain correct answer a.e.f.
(iii)	$\frac{1}{2}$	B1ft		S_∞ stated or start at n + 1 as in (ii)
	$\frac{1}{n+1} - \frac{1}{n+2}$	M1		S_∞ - their (ii) or show correct cancelling
	$\frac{1}{(n+1)(n+2)}$	A1	**3**	Obtain **given** answer correctly
		11		

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